Predictive control

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Outline

1 - Minimal variance control

$$J(u(t)) = \varepsilon\left(\left(y(t+d+1) - y^*(t+d+1)\right)^2\right)$$

2 - One step ahead predictive control

$$J(u(t)) = \varepsilon\left(\left(y(t+d+1) - y^*(t+d+1)\right)^2 + \mu(D(q^{-1})u(t))^2\right)$$

3 - One step ahead predictive control with frequency weighting

$$J(u(t)) = \varepsilon\left(\left(y(t+d+1) - y^*(t+d+1)\right)^2 + \mu\left(u_f(t)\right)^2\right)$$

$$u_f(t) = \frac{W(q^{-1})}{H(q^{-1})}u(t)$$

Minimal Variance Control

The criteria

Find the control value u(t) that minimizes the following criteria

$$J(u(t)) = \varepsilon\left(\left(y(t+d+1) - y^*(t+d+1)\right)^2\right)$$

Optimal prediction

$$\hat{y}(t+j/t) = \frac{F(q^{-1})}{C(q^{-1})}y(t) + \frac{B(q^{-1})E(q^{-1})D(q^{-1})}{C(q^{-1})}u(t+j-d-1)$$

$j = d+1$

$$\hat{y}(t+d+1/t) = \frac{F(q^{-1})}{C(q^{-1})}y(t) + \frac{B(q^{-1})E(q^{-1})D(q^{-1})}{C(q^{-1})}u(t)$$

$$C(q^{-1}) = A(q^{-1})D(q^{-1})E(q^{-1}) + q^{-d-1}F(q^{-1})$$

Derivation of the criteria

$$J(u(t)) = \varepsilon\left(\left(y(t+d+1) - y^*(t+d+1)\right)^2\right)$$

$$\hat{J}(u(t)) = \varepsilon\left(\left(\hat{y}(t+d+1/t) - y^*(t+d+1)\right)^2\right)$$

Derivation

$$\frac{\partial \hat{J}(u(t))}{\partial u(t)} = 2\left(\hat{y}(t+d+1/t) - y^*(t+d+1)\right)\frac{\partial \hat{y}(t+d+1/t)}{\partial u(t)}$$

$$\frac{\partial \hat{y}(t+d+1/t)}{\partial u(t)} = \frac{b_0 e_0 d_0}{c_0} = \frac{b_0 e_0}{c_0} = b_0$$

$$\frac{\partial \hat{J}(u(t))}{\partial u(t)} = 0 \Rightarrow \left(\hat{y}(t+d+1/t) - y^*(t+d+1)\right) = 0$$

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Linear Time Invariant controller structure

$$\frac{\partial \hat{J}\,(u(t))}{\partial u(t)} = 0 \Rightarrow \left(\hat{y}(t+d+1/t) - y^*(t+d+1)\right) = 0$$

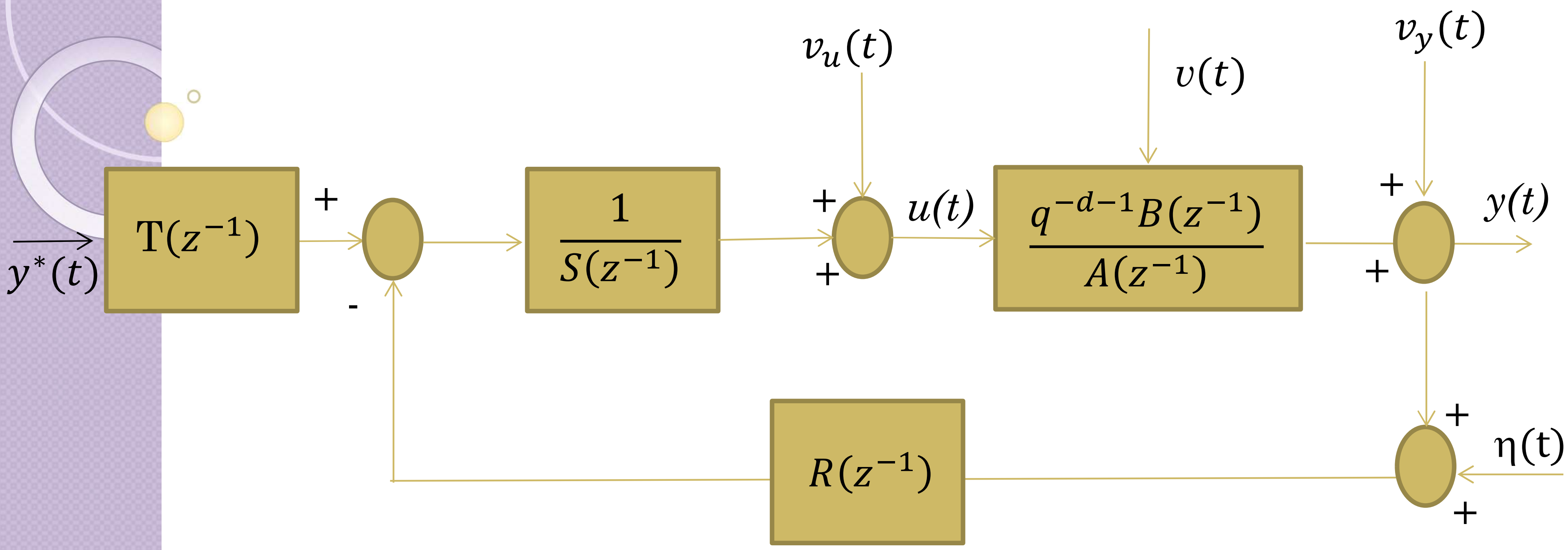
$$\Rightarrow \quad \hat{y}(t+d+1/t) = y^*(t+d+1)$$

$$\Rightarrow \quad \frac{F(q^{-1})}{C(q^{-1})} y(t) + \frac{B(q^{-1})E(q^{-1})D(q^{-1})}{C(q^{-1})} u(t) = y^*(t+d+1)$$

$$\Rightarrow \quad F(q^{-1})y(t) + B(q^{-1})E(q^{-1})D(q^{-1})u(t) = C(q^{-1})y^*(t+d+1)$$

$$\Rightarrow \quad \boxed{R(q^{-1})y(t) + S(q^{-1})u(t) = T(q^{-1})y^*(t+d+1)}$$

Closed-loop analysis



$v_u(t)$ *Input disturbance (low frequency)*

$v_y(t)$ *Output disturbance (low frequency)*

$y^*(t)$ *Reference sequence*

$\eta(t)$ *Noise mesurements (high frequency)*

Closed-loop performances

$$A(q^{-1})y(t) = q^{-d-1}B(q^{-1})\big(u(t)\big) + v(t)$$ *System equation*

$$R(q^{-1})y(t) + S(q^{-1})u(t) = T(q^{-1})y^*(t+d+1)$$ *Controller equation*

Output performances

$$y(t) = \frac{B(q^{-1})T(q^{-1})}{P_c(q^{-1})}y^*(t) + \frac{S(q^{-1})}{P_c(q^{-1})}v(t)$$

Input performances

$$u(t) = \frac{A(q^{-1})T(q^{-1})}{P_c(q^{-1})}y^*(t+d+1) - \frac{R(q^{-1})}{P_c(q^{-1})}v(t)$$

Characteristic polynomial

$$P_c(z^{-1}) = A(z^{-1})S(z^{-1}) + q^{-d-1}B(z^{-1})R(z^{-1})$$

$$= A(z^{-1})E(z^{-1})B(z^{-1})D(z^{-1}) + q^{-d-1}B(z^{-1})F(z^{-1})$$

$$= B(z^{-1})\left(A(z^{-1})E(z^{-1})D(z^{-1}) + q^{-d-1}F(z^{-1})\right)$$

$$= B(z^{-1})C(z^{-1})$$

!

$B(z^{-1})$ *and* $C(z^{-1})$ *MUST be stable polynomials (Hurwitz)*

Closed-loop performances

Output tracking performances

$$y(t) = \frac{B(q^{-1})T(q^{-1})}{P_c(q^{-1})} y^*(t) = \frac{B(q^{-1})C(q^{-1})}{B(q^{-1})C(q^{-1})} y^*(t) = y^*(t)$$

⇨ *Perfect tracking !!*

Disturbance rejection performances

$$y(t) = \frac{S(q^{-1})}{P_c(q^{-1})} v(t) = \frac{E(q^{-1})B(q^{-1})D(q^{-1})}{B(q^{-1})C(q^{-1})} v(t) = \frac{E(q^{-1})D(q^{-1})}{C(q^{-1})} v(t) = E(q^{-1})\gamma(t)$$

⇨ $y(t) - y^*(t) = E(q^{-1})\gamma(t) = \tilde{y}\,(t/t-d-1)$

Minimal Variance Control

Input tracking performances

$$u(t) = \frac{A(q^{-1})T(q^{-1})}{P_c(q^{-1})} y^*(t+d+1) = \frac{A(q^{-1})}{B(q^{-1})} y^*(t+d+1)$$

➡ *Inversion of the model!!*

➡ *High Energy consumption and input saturation problem*

Input rejection performances

➡ $$u(t) = -\frac{R(q^{-1})}{P_c(z^{-1})} v(t) = -\frac{F(q^{-1})}{B(z^{-1})C(z^{-1})} v(t)$$

One step ahead predictive control

The modified criteria

Find the control value u(t) that minimizes the following criteria

$$J(u(t)) = \varepsilon\left(\left(y(t+d+1) - y^*(t+d+1)\right)^2 + \mu\left(D(q^{-1})u(t)\right)^2\right)$$

Additionnal term $\triangleq$ *energy consumption term*

$\mu = 0 \Rightarrow$ *Minimal Variance Control*

$$\hat{J}(u(t)) = \left(\hat{y}(t+d+1/t) - y^*(t+d+1)\right)^2 + \mu\left(D(q^{-1})u(t)\right)^2$$

Derivation of the criteria

$$\hat{J}(u(t)) = \left(\hat{y}(t+d+1/t) - y^*(t+d+1)\right)^2 + \mu(D(q^{-1})u(t))^2$$

$$\Rightarrow \frac{\partial \hat{J}(u(t))}{\partial u(t)} = 2\left(\hat{y}(t+d+1/t) - y^*(t+d+1)\right)\frac{\partial \hat{y}(t+d+1/t)}{\partial u(t)} + 2\mu D(q^{-1})u(t)\frac{\partial (D(q^{-1})u(t))}{\partial u(t)}$$

$$\Rightarrow \frac{\partial \hat{y}(t+d+1/t)}{\partial u(t)} = \frac{b_0 e_0 d_0}{c_0} = b_0$$

$$\frac{\partial (D(q^{-1})u(t))}{\partial u(t)} = \frac{\partial (u(t) + d_1 u(t-1) + .. + d_{nd} u(t-n_d))}{\partial u(t)} = 1$$

Derivation of the criteria

$$\frac{\partial \hat{J}(u(t))}{\partial u(t)} = 2b_0\big(\hat{y}(t+d+1/t) - y^*(t+d+1)\big) + 2\mu D(q^{-1})u(t)$$

$$\frac{\partial \hat{J}(u(t))}{\partial u(t)} = 0 \Rightarrow D(q^{-1})u(t) = \frac{b_0}{\mu}\big(y^*(t+d+1) - \hat{y}(t+d+1/t)\big)$$

Let us introduce the prediction equation to replace $\hat{y}(t+d+1/t)$

$$\xrightarrow[C(q^{-1})]{\text{Operate by}} \quad C(q^{-1})D(q^{-1})u(t) = \frac{b_0}{\mu}\big(C(q^{-1})y^*(t+d+1) - C(q^{-1})\hat{y}(t+d+1/t)\big)$$

Linear Time Invariant controller structure

$$C(q^{-1})D(q^{-1})u(t) = \frac{b_0}{\mu}\left(C(q^{-1})y^*(t+d+1) - C(q^{-1})\hat{y}(t+d+1/t)\right)$$

Introduce the prediction equation

$$C(q^{-1})D(q^{-1})u(t) = \frac{b_0}{\mu}(C(q^{-1})y^*(t+d+1) - F(q^{-1})y(t) - E(q^{-1})B(q^{-1})D(q^{-1})u(t))$$

$$\left\{\frac{b_0}{\mu}E(q^{-1})B(q^{-1})D(q^{-1}) + C(q^{-1})D(q^{-1})\right\}u(t) + \frac{b_0}{\mu}F(q^{-1})y(t) = \frac{b_0}{\mu}C(q^{-1})y^*(t+d+1)$$

$$\boxed{S(q^{-1})u(t) + R(q^{-1})y(t) = T(q^{-1})y^*(t+d+1)}$$

Closed-loop performances

$$A(q^{-1})y(t) = q^{-d-1}B(q^{-1})\big(u(t)\big) + v(t)$$ *System equation*

$$R(q^{-1})y(t) + S(q^{-1})u(t) = T(q^{-1})y^*(t+d+1)$$ *Controller equation*

Output performances

$$y(t) = \frac{B(q^{-1})T(q^{-1})}{P_c(q^{-1})}y^*(t) + \frac{S(q^{-1})}{P_c(q^{-1})}v(t)$$

Input performances

$$u(t) = \frac{A(q^{-1})T(q^{-1})}{P_c(q^{-1})}y^*(t+d+1) - \frac{R(q^{-1})}{P_c(q^{-1})}v(t)$$

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Characteristic polynomial

$$P_c(z^{-1}) = A(z^{-1})S(z^{-1}) + q^{-d-1}B(z^{-1})R(z^{-1})$$

$$= A(z^{-1})\left\{\frac{b_0}{\mu}E(z^{-1})B(z^{-1})D(z^{-1}) + C(z^{-1})D(z^{-1})\right\}$$
$$+ q^{-d-1}B(z^{-1})\frac{b_0}{\mu}F(z^{-1})$$

$$= A(z^{-1})C(z^{-1})D(z^{-1}) + \frac{b_0}{\mu}B(z^{-1})\{A(z^{-1})E(z^{-1})D(z^{-1}) + q^{-d-1}F(z^{-1})\}$$

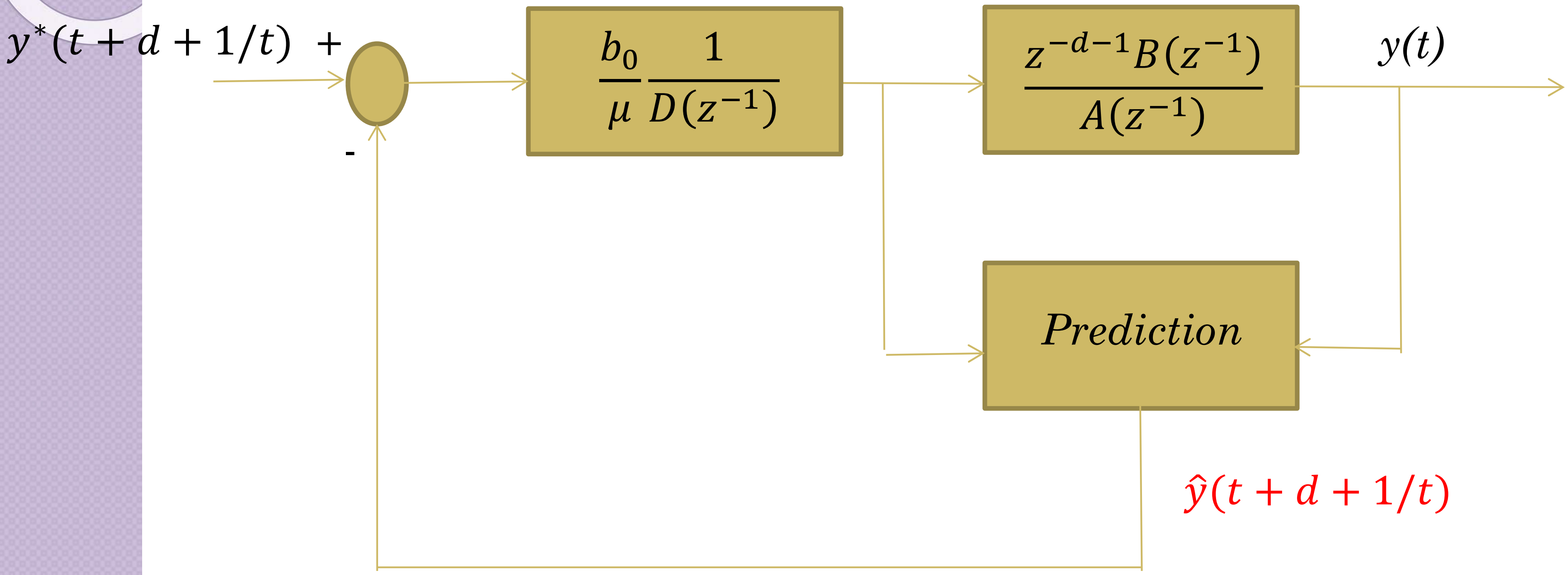
⇒ *Introduce the prediction equation*

$$P_c(z^{-1}) = A(z^{-1})C(z^{-1})D(z^{-1}) + \frac{b_0}{\mu}B(z^{-1})C(z^{-1})$$

$$P_c(z^{-1}) = C(z^{-1})\left\{A(z^{-1})D(z^{-1}) + \frac{b_0}{\mu}B(z^{-1})\right\}$$

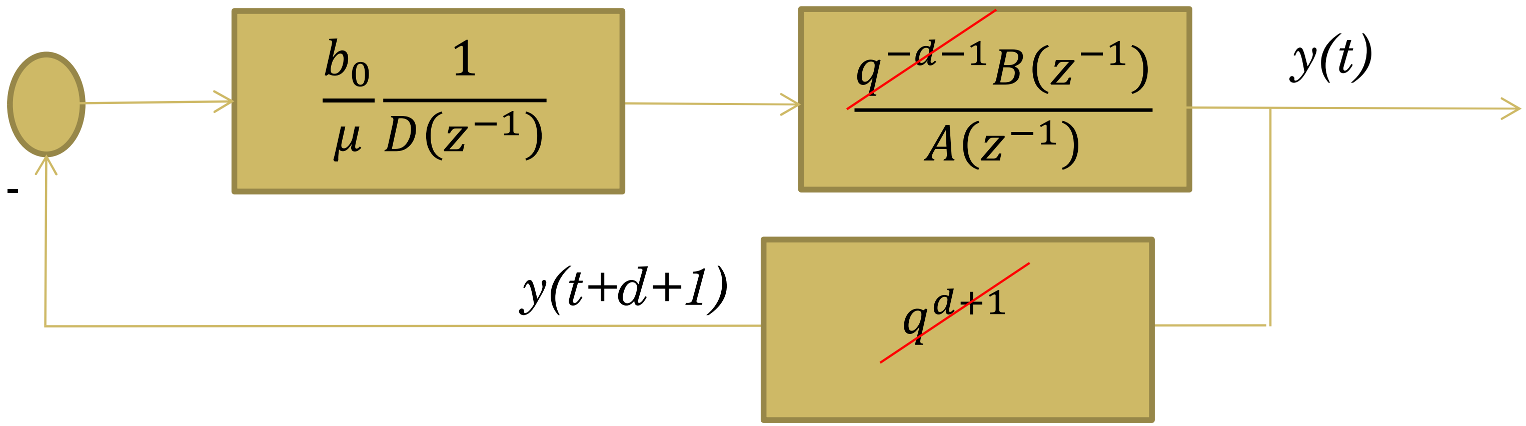
Equivalent implementation scheme

$$D(q^{-1})u(t) = \frac{b_0}{\mu}\left(y^*(t+d+1) - \hat{y}(t+d+1/t)\right)$$



Equivalent scheme for stability analysis

$$v(t) = 0 \Rightarrow \hat{y}(t+d+1/t) = y(t+d+1/t) = q^{d+1}y(t)$$



$$P_c(z^{-1}) = C(z^{-1})\left\{A(z^{-1})D(z^{-1}) + \frac{b_0}{\mu}B(z^{-1})\right\} \Rightarrow P_c(z^{-1}) = A(z^{-1})D(z^{-1}) + \frac{b_0}{\mu}B(z^{-1})$$

A single synthesis parameter : root-locus tool

Tracking performances

$$y(t)=\frac{B(q^{-1})T(q^{-1})}{P_c(q^{-1})}y^*(t)=\frac{\frac{b_0}{\mu}B(q^{-1})C(q^{-1})}{C(q^{-1})P_f(q^{-1})}y^*(t)=\frac{b_0}{\mu}\frac{B(q^{-1})}{P_f(q^{-1})}y^*(t)$$

$$\Rightarrow \frac{y(z^{-1})}{y^*(z^{-1})}=\frac{b_0}{\mu}\frac{B(z^{-1})}{P_f(z^{-1})}=\frac{b_0}{\mu}\frac{B(z^{-1})}{A(z^{-1})D(z^{-1})+\frac{b_0}{\mu}B(z^{-1})}$$

⇒ *Tracking dynamics does not depend on the predictor*

Static performances

No bias ⇒

$$\frac{b_0}{\mu}\frac{B(1)}{A(1)D(1)+\frac{b_0}{\mu}B(1)}=1\Rightarrow D(1)=0$$

$$\Rightarrow D(q^{-1})=(1-q^{-1})\mathrm{D}'(q^{-1})$$ *Integral action*

Disturbance rejection

$$y(t) = \frac{S(q^{-1})}{P_c(q^{-1})} v(t) \qquad S(q^{-1}) = D(q^{-1})\left(\frac{b_0}{\mu} E(q^{-1})B(q^{-1}) + C(q^{-1})\right)$$

$$y(t) = \frac{D(q^{-1})\left(\frac{b_0}{\mu} E(q^{-1})B(q^{-1}) + C(q^{-1})\right)}{C(q^{-1})P_f(q^{-1})} v(t)$$

$$= \frac{D(q^{-1})\left(\frac{b_0}{\mu} E(q^{-1})B(q^{-1}) + C(q^{-1})\right)}{C(q^{-1})P_f(q^{-1})} \frac{C(q^{-1})}{D(q^{-1})} \gamma(t)$$

$$= \frac{\left(\frac{b_0}{\mu} E(q^{-1})B(q^{-1}) + C(q^{-1})\right)}{P_f(q^{-1})} \gamma(t)$$

$P_f(q^{-1})$ *Hurwitz* ⇒ *Disturbance rejection*

One step ahead predictive control with input frequency weighting

The modified criteria

Find the control value u(t) that minimizes the following criteria

$$J(u(t)) = \varepsilon\left(\left(y(t+d+1) - y^*(t+d+1)\right)^2 + \mu\left(u_f(t)\right)^2\right)$$

with $u_f(t) = \frac{W(q^{-1})}{H(q^{-1})}u(t)$ $\mu = \frac{b_0 h_0}{w_0}$

$\frac{W(q^{-1})}{H(q^{-1})}$ *is called Input Frequency Weighting*

Derivation of the criteria

$$\hat{J}(u(t)) = \left(\hat{y}(t+d+1/t) - y^*(t+d+1)\right)^2 + \mu\left(u_f(t)\right)^2$$

$$\frac{\partial \hat{J}(u(t))}{\partial u(t)} = 2\left(\hat{y}(t+d+1/t) - y^*(t+d+1)\right)\frac{\partial \hat{y}(t+d+1/t)}{\partial u(t)}$$

$$+2\mu u_f(t)\frac{\partial\left(u_f(t)\right)}{\partial u(t)}$$

$$\frac{\partial \hat{y}(t+d+1/t)}{\partial u(t)} = \frac{b_0 e_0 d_0}{c_0} = b_0$$

$$\frac{\partial\left(u_f(t)\right)}{\partial u(t)} = \frac{w_0}{h_0}$$

Derivation of the criteria

$$\frac{\partial \hat{J}(u(t))}{\partial u(t)} = 2b_0\left(\hat{y}(t+d+1/t) - y^*(t+d+1)\right) + 2\mu\frac{w_0}{h_0}u_f(t)$$

$$\frac{\partial \hat{J}(u(t))}{\partial u(t)} = 0 \Rightarrow u_f(t) = \frac{b_0 h_0}{\mu w_0}\left(y^*(t+d+1) - \hat{y}(t+d+1/t)\right)$$

Let us introduce the prediction equation to replace $\hat{y}(t+d+1/t)$

Operate by $\xrightarrow{\hspace{1cm}}$ $C(q^{-1})$

$$C(q^{-1})u_f(t) = \left(C(q^{-1})y^*(t+d+1) - C(q^{-1})\hat{y}(t+d+1/t)\right)$$

Linear Time Invariant controller

$$C(q^{-1})u_f(t) = \{C(q^{-1})y^*(t+d+1) - F(q^{-1})y(t) - E(q^{-1})B(q^{-1})D(q^{-1})u(t)\}$$

Let us introduce the relation between $u_f(t)$ *and* $u(t)$

Operate by $\xrightarrow{}$ $H(q^{-1})$

$$C(q^{-1})H(q^{-1})u_f(t) = H(q^{-1})\{C(q^{-1})y^*(t+d+1) - F(q^{-1})y(t) - E(q^{-1})B(q^{-1})D(q^{-1})u(t)\}$$

$$C(q^{-1})W(q^{-1})u(t) = H(q^{-1})\{C(q^{-1})y^*(t+d+1) - F(q^{-1})y(t) - E(q^{-1})B(q^{-1})D(q^{-1})u(t)\}$$

$$\{C(q^{-1})W(q^{-1}) + H(q^{-1})E(q^{-1})B(q^{-1})D(q^{-1})\}u(t) + H(q^{-1})\,F(q^{-1})y(t) = H(q^{-1})C(q^{-1})y^*(t+d+1)$$

$$S(q^{-1})u(t) + R(q^{-1})y(t) = T(q^{-1})y^*(t+d+1)$$

Disturbance rejection

$$y(t) = \frac{S(q^{-1})}{P_c(q^{-1})} v(t)$$

$$S(q^{-1}) = C(q^{-1})W(q^{-1}) + H(q^{-1})E(q^{-1})B(q^{-1})D(q^{-1})$$

$$y(t) = \frac{C(q^{-1})W(q^{-1})+H(q^{-1})E(q^{-1})B(q^{-1})D(q^{-1})}{P_c(q^{-1})} v(t)$$

$$= \frac{C(q^{-1})W(q^{-1})+H(q^{-1})E(q^{-1})B(q^{-1})D(q^{-1})}{P_c(q^{-1})} \frac{C(q^{-1})}{D(q^{-1})} \gamma(t)$$

$P_c(q^{-1})$ *Hurwitz but* $D(q^{-1})$ *is not a Hurwitz polynomial*

Disturbance rejection $\Leftrightarrow$ $D(q^{-1})$ *factorizes* $S(q^{-1})$

$\Leftrightarrow$ $W(q^{-1}) = D(q^{-1})G(q^{-1})$

$$S(q^{-1}) = D(q^{-1})\{C(q^{-1})G(q^{-1}) + H(q^{-1})E(q^{-1})B(q^{-1})\}$$

Closed-loop performances

$$A(q^{-1})y(t) = q^{-d-1}B(q^{-1})\big(u(t)\big) + v(t)$$ *System equation*

$$R(q^{-1})y(t) + S(q^{-1})u(t) = T(q^{-1})y^*(t+d+1)$$ *Controller equation*

Output performances

$$y(t) = \frac{B(q^{-1})T(q^{-1})}{P_c(q^{-1})}y^*(t) + \frac{S(q^{-1})}{P_c(q^{-1})}v(t)$$

Input performances

$$u(t) = \frac{A(q^{-1})T(q^{-1})}{P_c(q^{-1})}y^*(t+d+1) - \frac{R(q^{-1})}{P_c(q^{-1})}v(t)$$

Closed-loop performances

$S(q^{-1}) = D(q^{-1})\{C(q^{-1})G(q^{-1}) + H(q^{-1})E(q^{-1})B(q^{-1})\}$

Characteristic polynomial

$$P_c(q^{-1}) = A(q^{-1})S(q^{-1}) + q^{-d-1}B(q^{-1})R(q^{-1})$$

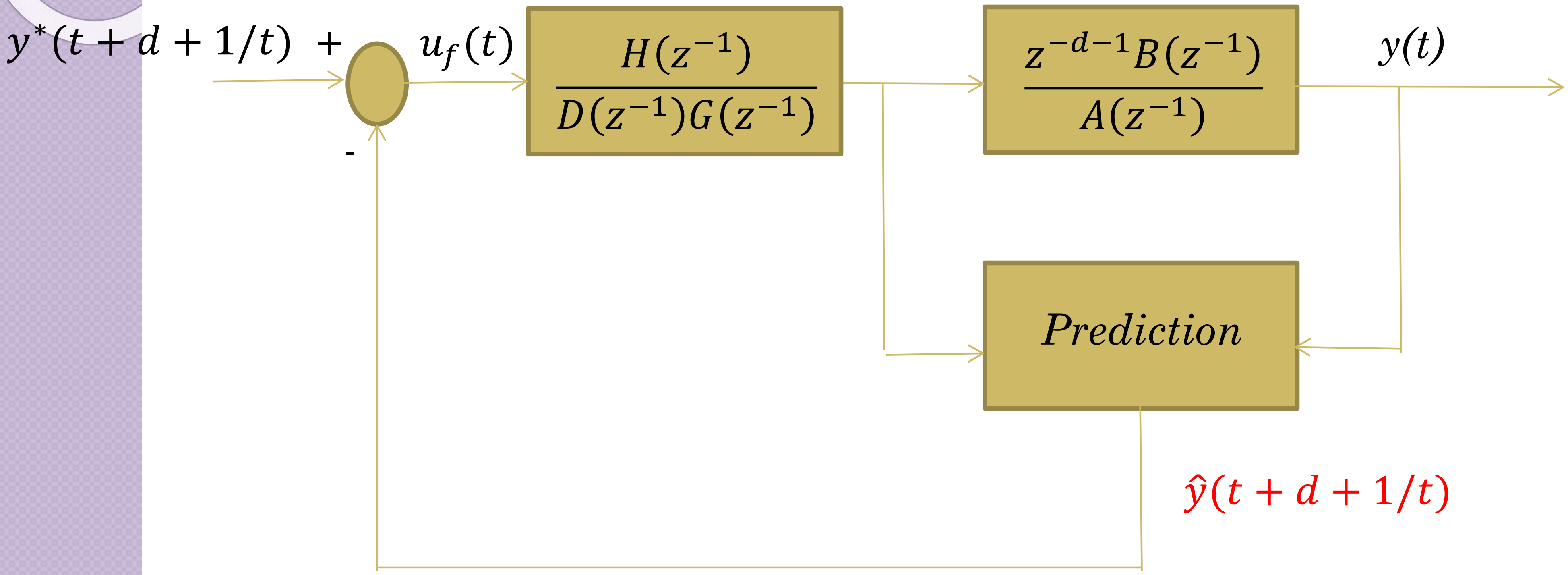
$$= A(q^{-1})D(q^{-1})\{C(q^{-1})G(q^{-1}) + H(q^{-1})E(q^{-1})B(q^{-1})\} + q^{-d-1}B(q^{-1})H(q^{-1})F(q^{-1})$$

$$= A(q^{-1})C(q^{-1})D(q^{-1})G(q^{-1}) + B(q^{-1})H(q^{-1})\{A(q^{-1})E(q^{-1})D(q^{-1}) + q^{-d-1}F(q^{-1})\}$$

$$\Rightarrow \quad P_c(q^{-1}) = C(q^{-1})\{A(q^{-1})D(q^{-1})G(q^{-1}) + B(q^{-1})H(q^{-1})\}$$

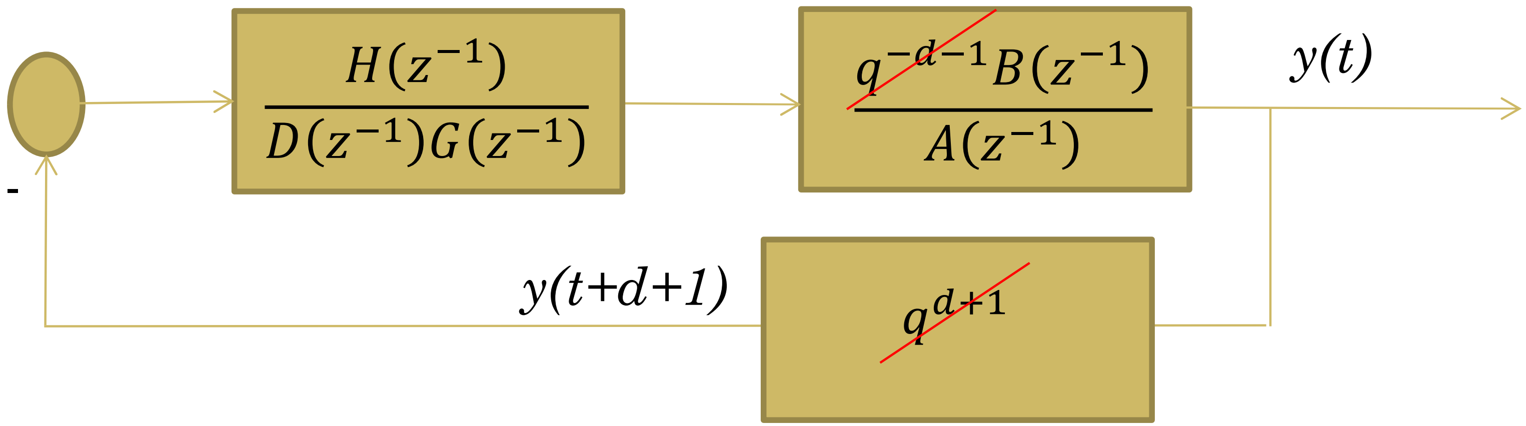
Equivalent implementation scheme

$$u_f(t) = \left(y^*(t+d+1) - \hat{y}(t+d+1/t)\right)$$



Equivalent scheme for stability analysis

$$v(t) = 0 \Rightarrow \hat{y}(t+d+1/t) = y(t+d+1/t) = q^{d+1}y(t)$$

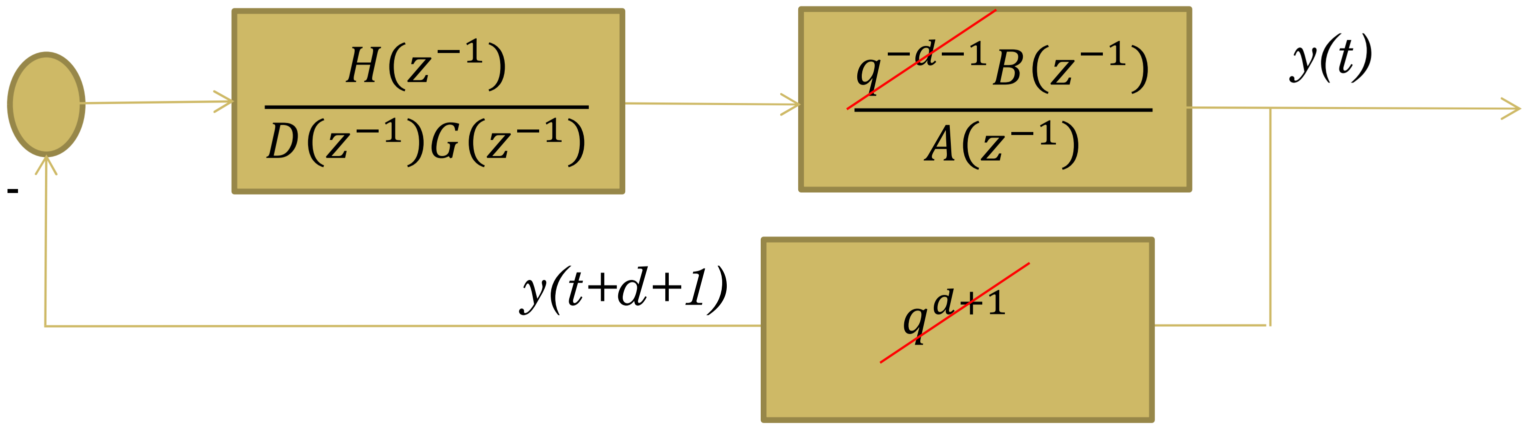


$$P_c(z^{-1}) = C(z^{-1})\{A(z^{-1})D(z^{-1})G(z^{-1}) + B(z^{-1})H(z^{-1})\}$$

Frequency weighting synthesis : pole placement or frequency design

Equivalent scheme for stability analysis

$$v(t) = 0 \Rightarrow \hat{y}(t+d+1/t) = y(t+d+1/t) = q^{d+1}y(t)$$



$$P_c(z^{-1}) = C(z^{-1})\{A(z^{-1})D(z^{-1})G(z^{-1}) + B(z^{-1})H(z^{-1})\}$$

Frequency weighting synthesis : pole placement or frequency design

Tracking performances

$$y(t) = \frac{B(q^{-1})T(q^{-1})}{P_c(q^{-1})} y^*(t) = \frac{B(q^{-1})H(q^{-1})C(q^{-1})}{C(q^{-1})(A(z^{-1})D(z^{-1})G(z^{-1}) + B(z^{-1})H(z^{-1}))} y^*(t)$$

$$\Rightarrow \quad \frac{y(z^{-1})}{y^*(z^{-1})} = \frac{B(z^{-1})H(z^{-1})}{A(z^{-1})D(z^{-1})G(z^{-1}) + B(z^{-1})H(z^{-1})}$$

$\Rightarrow$ *Tracking dynamics does not depend on the predictor*

Static performances

No bias $\Rightarrow$

$$\frac{B(1)H(1)}{A(1)D(1)G(1) + B(1)H(1)} = 0 \Rightarrow D(1) = 0$$

$$D(q^{-1}) = (1 - q^{-1})D'(q^{-1})$$

Integral action

Semi - Perfect and Perfect tracking

If one choses $T(q^{-1})$ *such that*

$$T(q^{-1}) = \frac{1}{B(1)} P_c(q^{-1})$$

$$\Rightarrow \quad \frac{y(z^{-1})}{y^*(z^{-1})} = \frac{B(z^{-1})T(z^{-1})}{P_c(z^{-1})} = \frac{B(z^{-1})}{B(1)} \quad \Rightarrow$$

Semi-perfect tracking

Moreover, if one choses $T(q^{-1})$ *such that*

$$A(z^{-1})D(z^{-1})G(z^{-1}) + B(z^{-1})H(z^{-1}) = B(z^{-1})M(z^{-1})$$

$$\mathrm{T}(z^{-1}) = M(z^{-1})$$

$$\Rightarrow \quad \frac{y(z^{-1})}{y^*(z^{-1})} = \frac{B(z^{-1})T(z^{-1})}{P_c(z^{-1})} = \frac{B(z^{-1})M(z^{-1})}{B(z^{-1})M(z^{-1})} = 1 \quad \Rightarrow$$

Perfect tracking

! *Perfect tracking IF AND ONLY IF* $\mathrm{B}(q^{-1})$ *HURWITZ*

Disturbance rejection

$$y(t) = \frac{S(q^{-1})}{P_c(q^{-1})} v(t) \qquad S(q^{-1}) = D(q^{-1})\left(\frac{b_0}{\mu} E(q^{-1})B(q^{-1}) + C(q^{-1})\right)$$

$$\Rightarrow y(t) = \frac{D(q^{-1})\left(\frac{b_0}{\mu}E(q^{-1})B(q^{-1}) + C(q^{-1})\right)}{C(q^{-1})P_f(q^{-1})} v(t)$$

$$= \frac{D(q^{-1})\left(\frac{b_0}{\mu}E(q^{-1})B(q^{-1}) + C(q^{-1})\right)}{C(q^{-1})P_f(q^{-1})} \frac{C(q^{-1})}{D(q^{-1})} \gamma(t)$$

$$= \frac{\left(\frac{b_0}{\mu}E(q^{-1})B(q^{-1}) + C(q^{-1})\right)}{P_f(q^{-1})} \gamma(t)$$

$\Rightarrow$ $P_f(q^{-1})$ *Hurwitz* $\Rightarrow$ *Disturbance rejection*