

Predictive control

Olivier Gehan
Ensicaen

olivier.gehan@ensicaen.fr



Outline

1 - Minimal variance control

$$J(u(t)) = \varepsilon \left((y(t+d+1) - y^*(t+d+1))^2 \right)$$

2 - One step ahead predictive control

$$J(u(t)) = \varepsilon \left((y(t+d+1) - y^*(t+d+1))^2 + \mu (D(q^{-1})u(t))^2 \right)$$

3 - One step ahead predictive control with frequency weighting

$$J(u(t)) = \varepsilon \left((y(t+d+1) - y^*(t+d+1))^2 + \mu (u_f(t))^2 \right)$$

$$u_f(t) = \frac{W(q^{-1})}{H(q^{-1})} u(t)$$



Minimal Variance Control

The criteria

Find the control value $u(t)$ that minimizes the following criteria

$$J(u(t)) = \varepsilon \left((y(t + d + 1) - y^*(t + d + 1))^2 \right)$$

Optimal prediction

$$\hat{y}(t + j/t) = \frac{F(q^{-1})}{C(q^{-1})} y(t) + \frac{B(q^{-1})E(q^{-1})D(q^{-1})}{C(q^{-1})} u(t + j - d - 1)$$

$$\downarrow j = d+1$$

$$\hat{y}(t + d + 1/t) = \frac{F(q^{-1})}{C(q^{-1})} y(t) + \frac{B(q^{-1})E(q^{-1})D(q^{-1})}{C(q^{-1})} u(t)$$

$$C(q^{-1}) = A(q^{-1})D(q^{-1})E(q^{-1}) + q^{-d-1}F(q^{-1})$$

Derivation of the criteria

$$J(u(t)) = \varepsilon \left((y(t + d + 1) - y^*(t + d + 1))^2 \right)$$

$$\hat{J}(u(t)) = \varepsilon \left((\hat{y}(t + d + 1/t) - y^*(t + d + 1))^2 \right)$$

Derivation

$$\frac{\partial \hat{J}(u(t))}{\partial u(t)} = 2(\hat{y}(t + d + 1/t) - y^*(t + d + 1)) \frac{\partial \hat{y}(t + d + 1/t)}{\partial u(t)}$$

$$\frac{\partial \hat{y}(t + d + 1/t)}{\partial u(t)} = \frac{b_0 e_0 c_0}{d_0} = b_0 c_0$$

$$\frac{\partial \hat{J}(u(t))}{\partial u(t)} = 0 \Rightarrow (\hat{y}(t + d + 1/t) - y^*(t + d + 1)) = 0$$

Linear Time Invariant controller structure

$$\frac{\partial \hat{J}(u(t))}{\partial u(t)} = 0 \Rightarrow (\hat{y}(t + d + 1/t) - y^*(t + d + 1)) = 0$$

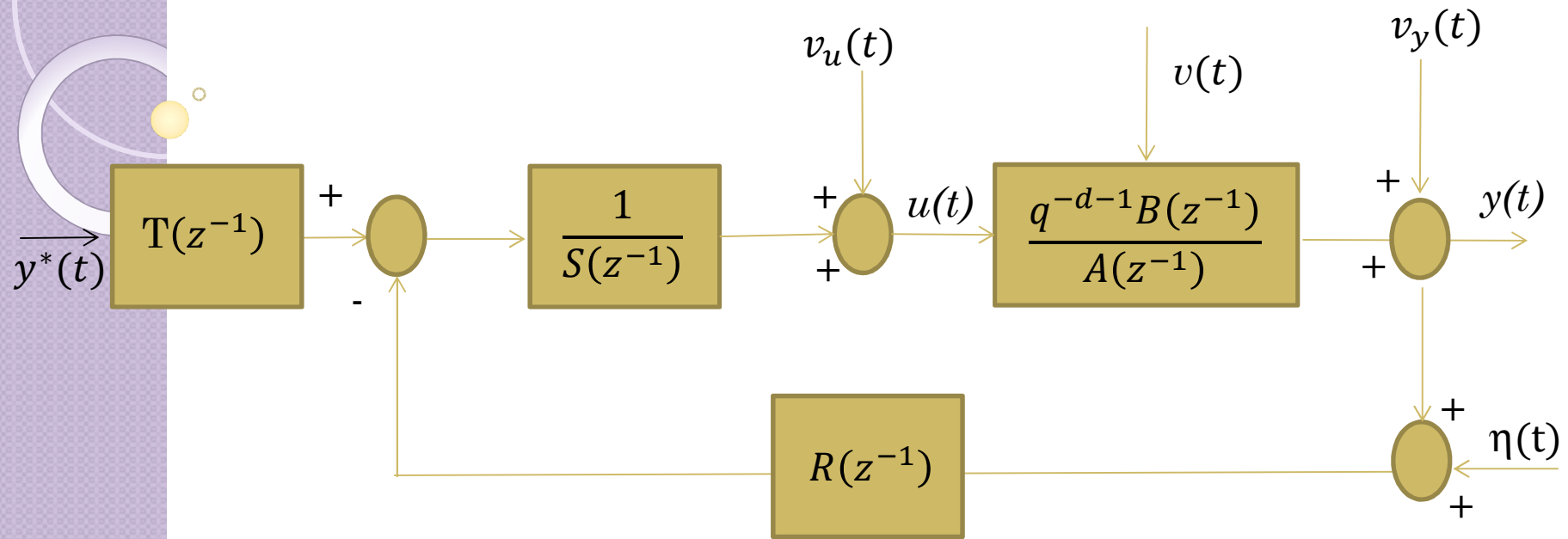
➔ $\hat{y}(t + d + 1/t) = y^*(t + d + 1)$

➔ $\frac{F(q^{-1})}{C(q^{-1})}y(t) + \frac{B(q^{-1})E(q^{-1})D(q^{-1})}{C(q^{-1})}u(t) = y^*(t + d + 1)$

➔ $F(q^{-1})y(t) + B(q^{-1})E(q^{-1})D(q^{-1})u(t) = C(q^{-1})y^*(t + d + 1)$

➔ $R(q^{-1})y(t) + S(q^{-1})u(t) = T(q^{-1})y^*(t + d + 1)$

Closed-loop analysis



$v_u(t)$ *Input disturbance (low frequency)*

$v_y(t)$ *Output disturbance (low frequency)*

$y^*(t)$ *Reference sequence*

$\eta(t)$ *Noise measurements (high frequency)*

Closed-loop performances

- $A(q^{-1})y(t) = q^{-d-1}B(q^{-1})(u(t)) + v(t)$ *System equation*
- $R(q^{-1})y(t) + S(q^{-1})u(t) = T(q^{-1})y^*(t + d + 1)$ *Controller equation*

Output performances

$$y(z^{-1}) = \frac{B(q^{-1})T(q^{-1})}{P_c(z^{-1})}y^*(t) + \frac{S(q^{-1})}{P_c(z^{-1})}v(t)$$

Input performances

$$u(z^{-1}) = \frac{A(q^{-1})T(q^{-1})}{P_c(z^{-1})}y^*(t + d + 1) - \frac{R(q^{-1})}{P_c(z^{-1})}v(t)$$

Closed-loop performances

Characteristic polynomial

$$\begin{aligned}P_c(z^{-1}) &= A(z^{-1})S(z^{-1}) + q^{-d-1}B(z^{-1})R(z^{-1}) \\&= A(z^{-1})E(z^{-1})B(z^{-1})D(z^{-1}) + q^{-d-1}B(z^{-1})F(z^{-1}) \\&= B(z^{-1})\left(A(z^{-1})E(z^{-1})E(z^{-1}) + q^{-d-1}F(z^{-1})\right) \\&= B(z^{-1})C(z^{-1})\end{aligned}$$

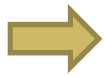


$B(z^{-1})$ *and* $C(z^{-1})$ **MUST** be stable polynomials (Hurwitz)

Closed-loop performances

Output tracking performances

$$y(t) = \frac{B(q^{-1})T(q^{-1})}{P_c(q^{-1})} y^*(t) = \frac{B(q^{-1})C(q^{-1})}{B(q^{-1})C(q^{-1})} y^*(t) = y^*(t)$$



Perfect tracking !!

Disturbance rejection performances

$$y(t) = \frac{S(q^{-1})}{P_c(q^{-1})} v(t) = \frac{E(q^{-1})B(q^{-1})D(q^{-1})}{B(q^{-1})C(q^{-1})} v(t) = \frac{E(q^{-1})D(q^{-1})}{C(q^{-1})} v(t) = E(q^{-1})\gamma(t)$$



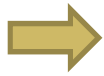
$$y(t) - y^*(t) = E(q^{-1})\gamma(t) = \tilde{y}(t/t - d - 1)$$

Minimal Variance Control

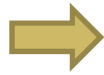
Closed-loop performances

Input tracking performances

$$u(t) = \frac{A(q^{-1})T(q^{-1})}{P_c(q^{-1})} y^*(t + d + 1) = \frac{A(q^{-1})}{B(q^{-1})} y^*(t + d + 1)$$



Inversion of the model!!



High Energy consumption and input saturation problem

Input rejection performances

$$\Rightarrow u(t) = -\frac{R(q^{-1})}{P_c(z^{-1})} v(t) = -\frac{F(q^{-1})}{B(z^{-1})C(z^{-1})} v(t)$$



One step ahead predictive control

The modified criteria

Find the control value $u(t)$ that minimizes the following criteria

$$J(u(t)) = \varepsilon \left((y(t+d+1) - y^*(t+d+1))^2 + \mu (D(q^{-1})u(t))^2 \right)$$

Additional term \triangleq energy consumption term

$\mu = 0 \Rightarrow$ Minimal Variance Control

$$\hat{J}(u(t)) = (\hat{y}(t+d+1/t) - y^*(t+d+1))^2 + \mu (D(q^{-1})u(t))^2$$

Derivation of the criteria

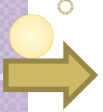
$$\hat{J}(u(t)) = (\hat{y}(t + d + 1/t) - y^*(t + d + 1))^2 + \mu(D(q^{-1})u(t))^2$$

$$\begin{aligned} \frac{\partial \hat{J}(u(t))}{\partial u(t)} &= 2(\hat{y}(t + d + 1/t) - y^*(t + d + 1)) \frac{\partial \hat{y}(t + d + 1/t)}{\partial u(t)} \\ &\quad + 2\mu D(q^{-1})u(t) \frac{\partial (D(q^{-1})u(t))}{\partial u(t)} \end{aligned}$$


$$\frac{\partial \hat{y}(t + d + 1/t)}{\partial u(t)} = \frac{b_0 e_0 d_0}{c_0} = b_0$$

$$\frac{\partial (D(q^{-1})u(t))}{\partial u(t)} = \frac{\partial (u(t) + d_1 q^{-1} + \dots + d_{nd} q^{-nd})}{\partial u(t)} = 1$$

Derivation of the criteria


$$\frac{\partial \hat{J}(u(t))}{\partial u(t)} = 2b_0(\hat{y}(t+d+1/t) - y^*(t+d+1)) + 2\mu D(q^{-1})u(t)$$

$$\frac{\partial \hat{J}(u(t))}{\partial u(t)} = 0 \Rightarrow D(q^{-1})u(t) = \frac{b_0}{\mu} (y^*(t+d+1) - \hat{y}(t+d+1/t))$$



Let us introduce the prediction equation to replace $\hat{y}(t+d+1/t)$

Operate by $\xrightarrow{C(q^{-1})}$

$$C(q^{-1})D(q^{-1})u(t) = \frac{b_0}{\mu} (C(q^{-1})y^*(t+d+1) - C(q^{-1})\hat{y}(t+d+1/t))$$

Linear Time Invariant controller structure

$$C(q^{-1})D(q^{-1})u(t) = \frac{b_0}{\mu} (C(q^{-1})y^*(t+d+1) - C(q^{-1})\hat{y}(t+d+1/t))$$

Introduce the prediction equation

$$C(q^{-1})D(q^{-1})u(t) = \frac{b_0}{\mu} (C(q^{-1})y^*(t+d+1) - F(q^{-1})y(t) - E(q^{-1})B(q^{-1})D(q^{-1})u(t))$$

$$\left\{ \frac{b_0}{\mu} E(q^{-1})B(q^{-1})D(q^{-1}) + C(q^{-1})D(q^{-1}) \right\} u(t) + \frac{b_0}{\mu} F(q^{-1})y(t) = \frac{b_0}{\mu} C(q^{-1})y^*(t+d+1)$$

$$S(q^{-1})u(t) + R(q^{-1})y(t) = T(q^{-1})y^*(t+d+1)$$

Closed-loop performances

- $A(q^{-1})y(t) = q^{-d-1}B(q^{-1})(u(t)) + v(t)$ *System equation*
- $R(q^{-1})y(t) + S(q^{-1})u(t) = T(q^{-1})y^*(t + d + 1)$ *Controller equation*

Output performances

$$y(z^{-1}) = \frac{B(q^{-1})T(q^{-1})}{P_c(z^{-1})}y^*(t) + \frac{S(q^{-1})}{P_c(z^{-1})}v(t)$$

Input performances

$$u(z^{-1}) = \frac{A(q^{-1})T(q^{-1})}{P_c(z^{-1})}y^*(t + d + 1) - \frac{R(q^{-1})}{P_c(z^{-1})}v(t)$$

Closed-loop performances

Characteristic polynomial

$$P_c(z^{-1}) = A(z^{-1})S(z^{-1}) + q^{-d-1}B(z^{-1})R(z^{-1})$$

$$= A(z^{-1}) \left\{ \frac{b_0}{\mu} E(z^{-1})B(z^{-1})D(z^{-1}) + C(z^{-1})D(z^{-1}) \right\} \\ + q^{-d-1}B(z^{-1}) \frac{b_0}{\mu} F(z^{-1})$$

$$= A(z^{-1})C(z^{-1})D(z^{-1}) + \frac{b_0}{\mu} B(z^{-1}) \{ A(z^{-1})E(z^{-1})D(z^{-1}) + q^{-d-1}F(z^{-1}) \}$$

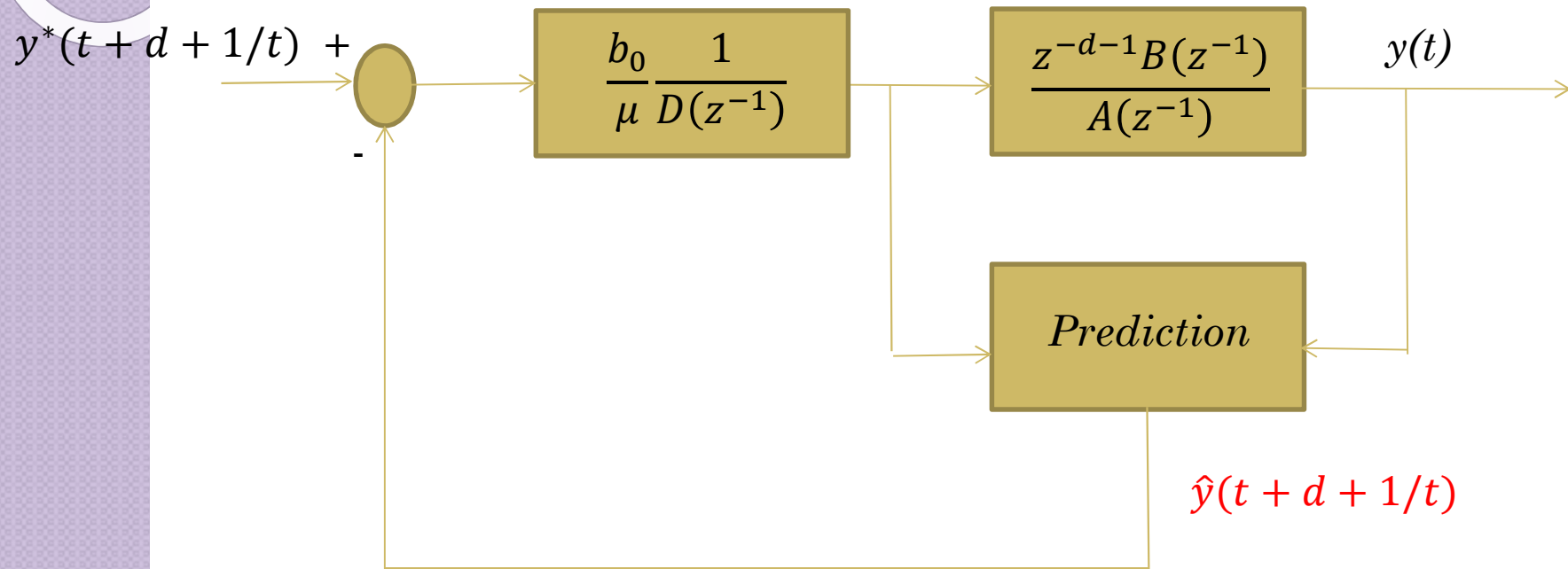
➔ Introduce the prediction equation

$$P_c(z^{-1}) = A(z^{-1})C(z^{-1})D(z^{-1}) + \frac{b_0}{\mu} B(z^{-1})C(z^{-1})$$

$$P_c(z^{-1}) = C(z^{-1}) \left\{ A(z^{-1})D(z^{-1}) + \frac{b_0}{\mu} B(z^{-1}) \right\}$$

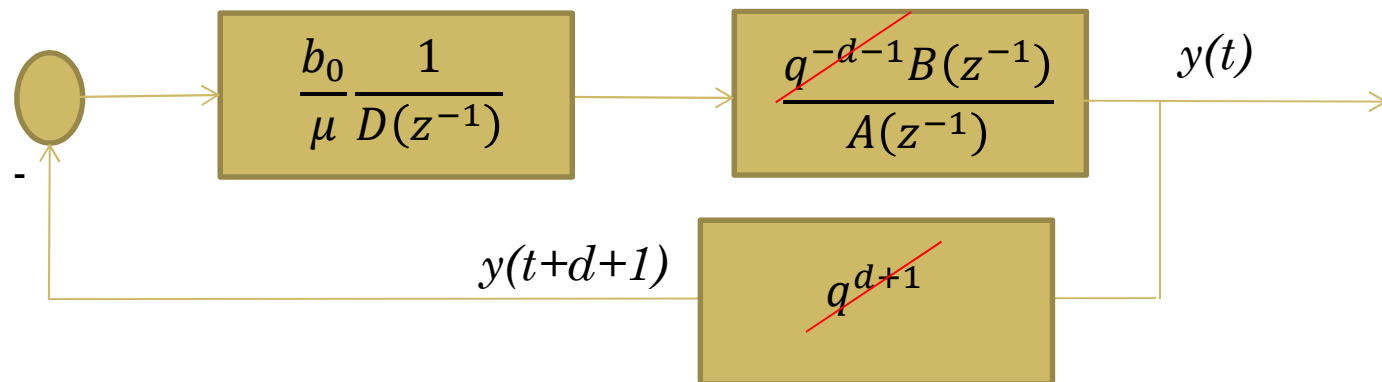
Equivalent implementation scheme

$$D(q^{-1})u(t) = \frac{b_0}{\mu} (y^*(t+d+1) - \hat{y}(t+d+1/t))$$



Equivalent scheme for stability analysis

$$v(t) = 0 \Rightarrow \hat{y}(t + d + 1/t) = y(t + d + 1/t) = q^{d+1}y(t)$$



$$P_c(z^{-1}) = C(z^{-1}) \left\{ A(z^{-1})D(z^{-1}) + \frac{b_0}{\mu} B(z^{-1}) \right\} \Rightarrow P_c(z^{-1}) = A(z^{-1})D(z^{-1}) + \frac{b_0}{\mu} B(z^{-1})$$

A single synthesis parameter : root-locus tool

Tracking performances

$$y(t) = \frac{B(q^{-1})T(q^{-1})}{P_c(q^{-1})} y^*(t) = \frac{\frac{b_0}{\mu} B(q^{-1})C(q^{-1})}{C(q^{-1})P_f(q^{-1})} y^*(t) = \frac{b_0}{\mu} \frac{B(q^{-1})}{P_f(q^{-1})} y^*(t)$$

$$\frac{y(z^{-1})}{y^*(z^{-1})} = \frac{b_0}{\mu} \frac{B(z^{-1})}{P_f(z^{-1})} = \frac{b_0}{\mu} \frac{B(z^{-1})}{A(z^{-1})D(z^{-1}) + B(z^{-1})}$$

Tracking dynamics does not depend on the predictor

Static performances

No bias $\Rightarrow \frac{B(0)}{A(0)D(0) + B(0)} = 0 \Rightarrow D(0) = 0$

$$\Rightarrow D(q^{-1}) = (1 - q^{-1})D'(q^{-1}) \quad \text{Integral action}$$

Disturbance rejection

$$y(t) = \frac{S(q^{-1})}{P_c(q^{-1})} v(t)$$

$$S(q^{-1}) = D(q^{-1}) \left(\frac{b_0}{\mu} E(q^{-1}) B(q^{-1}) + C(q^{-1}) \right)$$

$$y(t) = \frac{D(q^{-1}) \left(\frac{b_0}{\mu} E(q^{-1}) B(q^{-1}) + C(q^{-1}) \right)}{C(q^{-1}) P_f(q^{-1})} v(t)$$

$$= \frac{D(q^{-1}) \left(\frac{b_0}{\mu} E(q^{-1}) B(q^{-1}) + C(q^{-1}) \right)}{C(q^{-1}) P_f(q^{-1})} \frac{C(q^{-1})}{D(q^{-1})} \gamma(t)$$

$$= \frac{\left(\frac{b_0}{\mu} E(q^{-1}) B(q^{-1}) + C(q^{-1}) \right)}{P_f(q^{-1})} \gamma(t)$$

$P_f(q^{-1})$ Hurwitz \Rightarrow Disturbance rejection



*One step ahead predictive control
with input frequency weighting*

The modified criteria

Find the control value $u(t)$ that minimizes the following criteria

$$J(u(t)) = \varepsilon \left((y(t + d + 1) - y^*(t + d + 1))^2 + \mu (u_f(t))^2 \right)$$

with

$$u_f(t) = \frac{W(q^{-1})}{H(q^{-1})} u(t) \quad \mu = \frac{b_0 h_0}{w_0}$$

$\frac{W(q^{-1})}{H(q^{-1})}$ is called *Input Frequency Weighting*

Derivation of the criteria

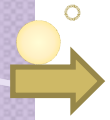
$$\hat{J}(u(t)) = (\hat{y}(t + d + 1/t) - y^*(t + d + 1))^2 + \mu(u_f(t))^2$$

$$\begin{aligned} \frac{\partial \hat{J}(u(t))}{\partial u(t)} &= 2(\hat{y}(t + d + 1/t) - y^*(t + d + 1)) \frac{\partial \hat{y}(t + d + 1/t)}{\partial u(t)} \\ &\quad + 2\mu u_f(t) \frac{\partial(u_f(t))}{\partial u(t)} \end{aligned}$$

$$\frac{\partial \hat{y}(t + d + 1/t)}{\partial u(t)} = \frac{b_0 e_0 d_0}{c_0} = b_0$$

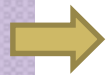
$$\frac{\partial(u_f(t))}{\partial u(t)} = \frac{w_0}{h_0}$$

Derivation of the criteria



$$\frac{\partial \hat{J}(u(t))}{\partial u(t)} = 2b_0(\hat{y}(t+d+1/t) - y^*(t+d+1)) + 2\mu \frac{w_0}{h_0} u_f(t)$$

$$\frac{\partial \hat{J}(u(t))}{\partial u(t)} = 0 \Rightarrow u_f(t) = \frac{b_0 h_0}{\mu w_0} (y^*(t+d+1) - \hat{y}(t+d+1/t))$$



Let us introduce the prediction equation to replace $\hat{y}(t+d+1/t)$

Operate by
 $\xrightarrow{C(q^{-1})}$

$$C(q^{-1})u_f(t) = (C(q^{-1})y^*(t+d+1) - C(q^{-1})\hat{y}(t+d+1/t))$$

Linear Time Invariant controller

$$C(q^{-1})u_f(t) = \{C(q^{-1})y^*(t + d + 1) - F(q^{-1})y(t) - E(q^{-1})B(q^{-1})D(q^{-1})u(t)\}$$

➔ Let us introduce the relation between $u_f(t)$ and $u(t)$

Operate by $\xrightarrow{H(q^{-1})}$

$$C(q^{-1})H(q^{-1})u_f(t) = H(q^{-1})\{C(q^{-1})y^*(t + d + 1) - F(q^{-1})y(t) - E(q^{-1})B(q^{-1})D(q^{-1})u(t)\}$$

$$C(q^{-1})W(q^{-1})u(t) = H(q^{-1})\{C(q^{-1})y^*(t + d + 1) - F(q^{-1})y(t) - E(q^{-1})B(q^{-1})D(q^{-1})u(t)\}$$

➔ $\{C(q^{-1})W(q^{-1}) + H(q^{-1})E(q^{-1})B(q^{-1})D(q^{-1})\}u(t) + H(q^{-1})F(q^{-1})y(t) = H(q^{-1})C(q^{-1})y^*(t + d + 1)$

$$S(q^{-1})u(t) + R(q^{-1})y(t) = T(q^{-1})y^*(t + d + 1)$$

Disturbance rejection

$$y(t) = \frac{S(q^{-1})}{P_c(q^{-1})} v(t)$$

$$S(q^{-1}) = C(q^{-1})W(q^{-1}) + H(q^{-1})E(q^{-1})B(q^{-1})D(q^{-1})$$

$$y(t) = \frac{C(q^{-1})W(q^{-1}) + H(q^{-1})E(q^{-1})B(q^{-1})D(q^{-1})}{P_c(q^{-1})} v(t)$$

$$= \frac{C(q^{-1})W(q^{-1}) + H(q^{-1})E(q^{-1})B(q^{-1})D(q^{-1})}{P_c(q^{-1})} \frac{C(q^{-1})}{D(q^{-1})} \gamma(t)$$

$P_c(q^{-1})$ Hurwitz but $D(q^{-1})$ is not a Hurwitz polynomial

Disturbance rejection $\Leftrightarrow D(q^{-1})$ factorizes $S(q^{-1})$

$$\Leftrightarrow W(q^{-1}) = D(q^{-1})G(q^{-1})$$

$$S(q^{-1}) = D(q^{-1})\{C(q^{-1})G(q^{-1}) + H(q^{-1})E(q^{-1})B(q^{-1})\}$$

Closed-loop performances

- $A(q^{-1})y(t) = q^{-d-1}B(q^{-1})(u(t)) + v(t)$ *System equation*
- $R(q^{-1})y(t) + S(q^{-1})u(t) = T(q^{-1})y^*(t + d + 1)$ *Controller equation*

Output performances

$$y(t) = \frac{B(q^{-1})T(q^{-1})}{P_c(q^{-1})}y^*(t) + \frac{S(q^{-1})}{P_c(q^{-1})}v(t)$$

Input performances

$$u(t) = \frac{A(q^{-1})T(q^{-1})}{P_c(q^{-1})}y^*(t + d + 1) - \frac{R(q^{-1})}{P_c(q^{-1})}v(t)$$

Closed-loop performances

Characteristic polynomial

$$S(q^{-1}) = D(q^{-1})\{C(q^{-1})G(q^{-1}) + H(q^{-1})E(q^{-1})B(q^{-1})\}$$

$$P_c(q^{-1}) = A(q^{-1})S(q^{-1}) + q^{-d-1}B(q^{-1})R(q^{-1})$$

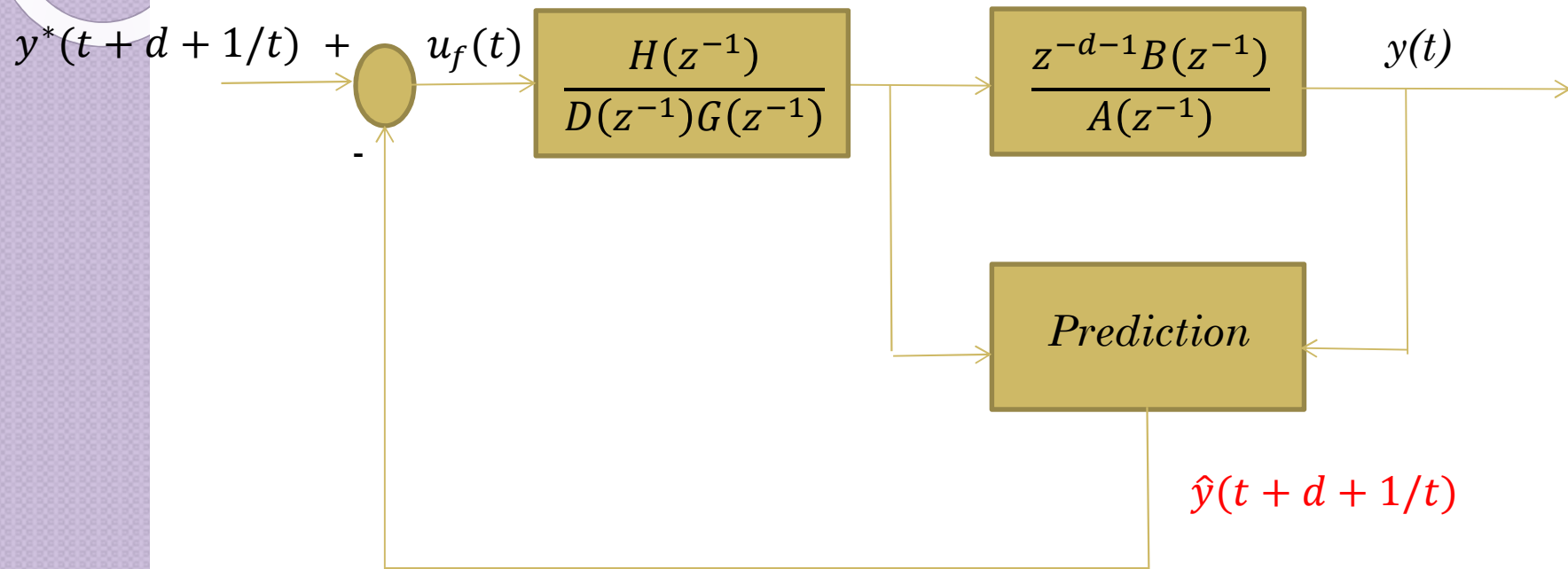
$$= A(q^{-1})D(q^{-1})\{C(q^{-1})G(q^{-1}) + H(q^{-1})E(q^{-1})B(q^{-1})\} + q^{-d-1}B(q^{-1})H(q^{-1})F(q^{-1})$$

$$= A(q^{-1})C(q^{-1})D(q^{-1})G(q^{-1}) + B(q^{-1})H(q^{-1})\{A(q^{-1})E(q^{-1})D(q^{-1}) + q^{-d-1}F(q^{-1})\}$$

$$\Rightarrow P_c(q^{-1}) = C(q^{-1})\{A(q^{-1})D(q^{-1})G(q^{-1}) + B(q^{-1})H(q^{-1})\}$$

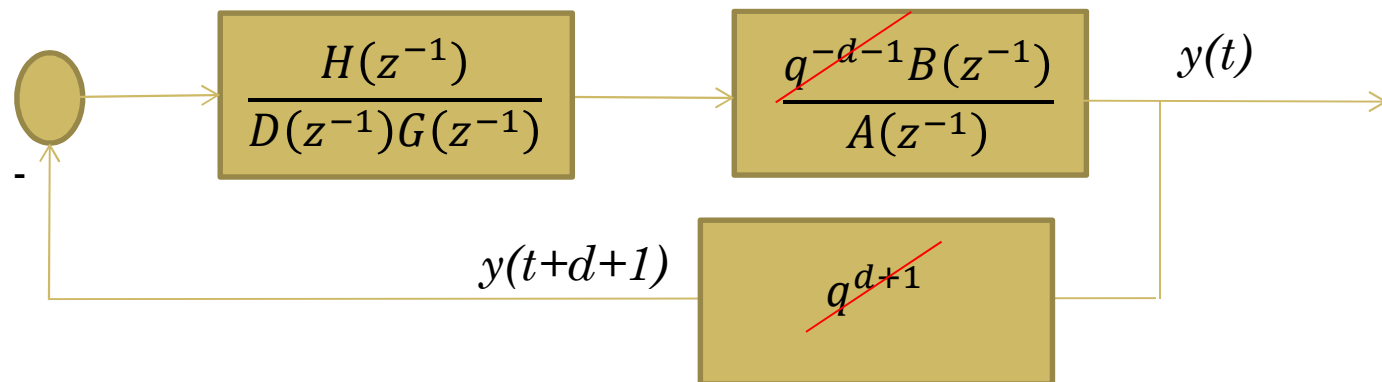
Equivalent implementation scheme

$$u_f(t) = (y^*(t + d + 1) - \hat{y}(t + d + 1/t))$$



Equivalent scheme for stability analysis

$$v(t) = 0 \Rightarrow \hat{y}(t + d + 1/t) = y(t + d + 1/t) = q^{d+1}y(t)$$

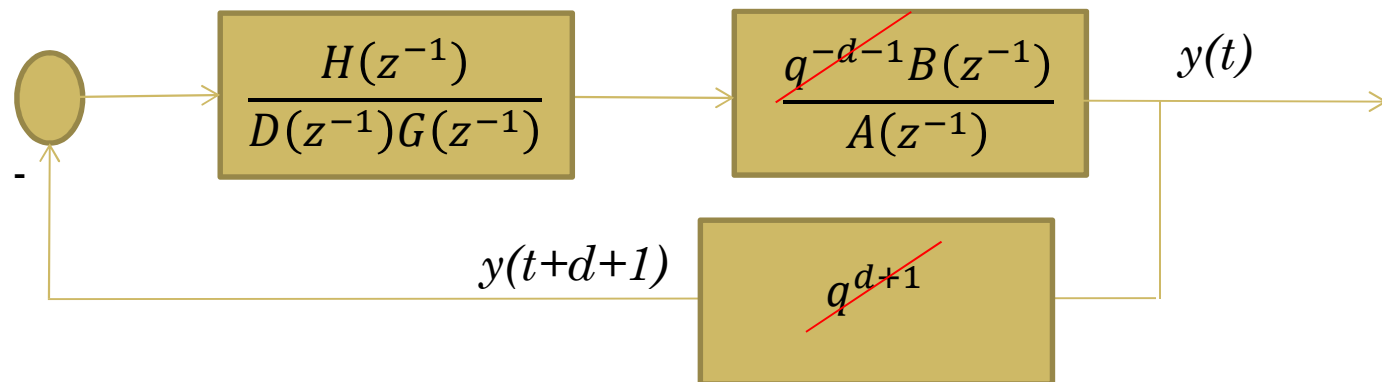


$$P_c(z^{-1}) = C(z^{-1})\{A(z^{-1})D(z^{-1})G(z^{-1}) + B(z^{-1})H(z^{-1})\}$$

Frequency weighting synthesis : pole placement or frequency design

Equivalent scheme for stability analysis

$$v(t) = 0 \Rightarrow \hat{y}(t + d + 1/t) = y(t + d + 1/t) = q^{d+1}y(t)$$



$$P_c(z^{-1}) = C(z^{-1})\{A(z^{-1})D(z^{-1})G(z^{-1}) + B(z^{-1})H(z^{-1})\}$$

Frequency weighting synthesis : pole placement or frequency design

Tracking performances

$$y(t) = \frac{B(q^{-1})T(q^{-1})}{P_c(q^{-1})} y^*(t) = \frac{B(q^{-1})H(q^{-1})C(q^{-1})}{C(q^{-1})A(z^{-1})D(z^{-1})G(z^{-1}) + B(z^{-1})H(z^{-1})} y^*(t)$$

$$\frac{y(z^{-1})}{y^*(z^{-1})} = \frac{B(z^{-1})H(z^{-1})}{A(z^{-1})D(z^{-1})G(z^{-1}) + B(z^{-1})H(z^{-1})}$$

Tracking dynamics does not depend on the predictor

Static performances

No bias $\Rightarrow \frac{B(1)H(1)}{A(1)D(1)G(1) + B(1)H(1)} = 0 \Rightarrow D(1) = 0$

$$D(q^{-1}) = (1 - q^{-1})D'(q^{-1}) \quad \text{Integral action}$$

Semi - Perfect and Perfect tracking

If one choses $T(q^{-1})$ such that

$$T(q^{-1}) = \frac{1}{B(1)} P_c(q^{-1})$$

$$\Rightarrow \frac{y(z^{-1})}{y^*(z^{-1})} = \frac{B(z^{-1})T(z^{-1})}{P_c(z^{-1})} = \frac{B(z^{-1})}{B(1)} \Rightarrow \text{Semi-perfect tracking}$$

Moreover, if one choses $T(q^{-1})$ such that

$$A(z^{-1})D(z^{-1})G(z^{-1}) + B(z^{-1})H(z^{-1}) = B(z^{-1})M(z^{-1})$$

$$T(z^{-1}) = M(z^{-1})$$

$$\Rightarrow \frac{y(z^{-1})}{y^*(z^{-1})} = \frac{B(z^{-1})T(z^{-1})}{P_c(z^{-1})} = \frac{B(z^{-1})M(z^{-1})}{B(z^{-1})M(z^{-1})} = 1 \Rightarrow \text{Perfect tracking}$$



Perfect tracking IF AND ONLY IF $B(q^{-1})$ HURWITZ

Disturbance rejection

$$y(t) = \frac{S(q^{-1})}{P_c(q^{-1})} v(t)$$

$$S(q^{-1}) = D(q^{-1}) \left(\frac{b_0}{\mu} E(q^{-1}) B(q^{-1}) + C(q^{-1}) \right)$$

$$y(t) = \frac{D(q^{-1}) \left(\frac{b_0}{\mu} E(q^{-1}) B(q^{-1}) + C(q^{-1}) \right)}{C(q^{-1}) P_f(q^{-1})} v(t)$$

$$= \frac{D(q^{-1}) \left(\frac{b_0}{\mu} E(q^{-1}) B(q^{-1}) + C(q^{-1}) \right)}{C(q^{-1}) P_f(q^{-1})} \frac{C(q^{-1})}{D(q^{-1})} \gamma(t)$$

$$= \frac{\left(\frac{b_0}{\mu} E(q^{-1}) B(q^{-1}) + C(q^{-1}) \right)}{P_f(q^{-1})} \gamma(t)$$

$P_f(q^{-1})$ Hurwitz \Rightarrow Disturbance rejection

Exercice on Matlab / Simulink

